Interface data transfer

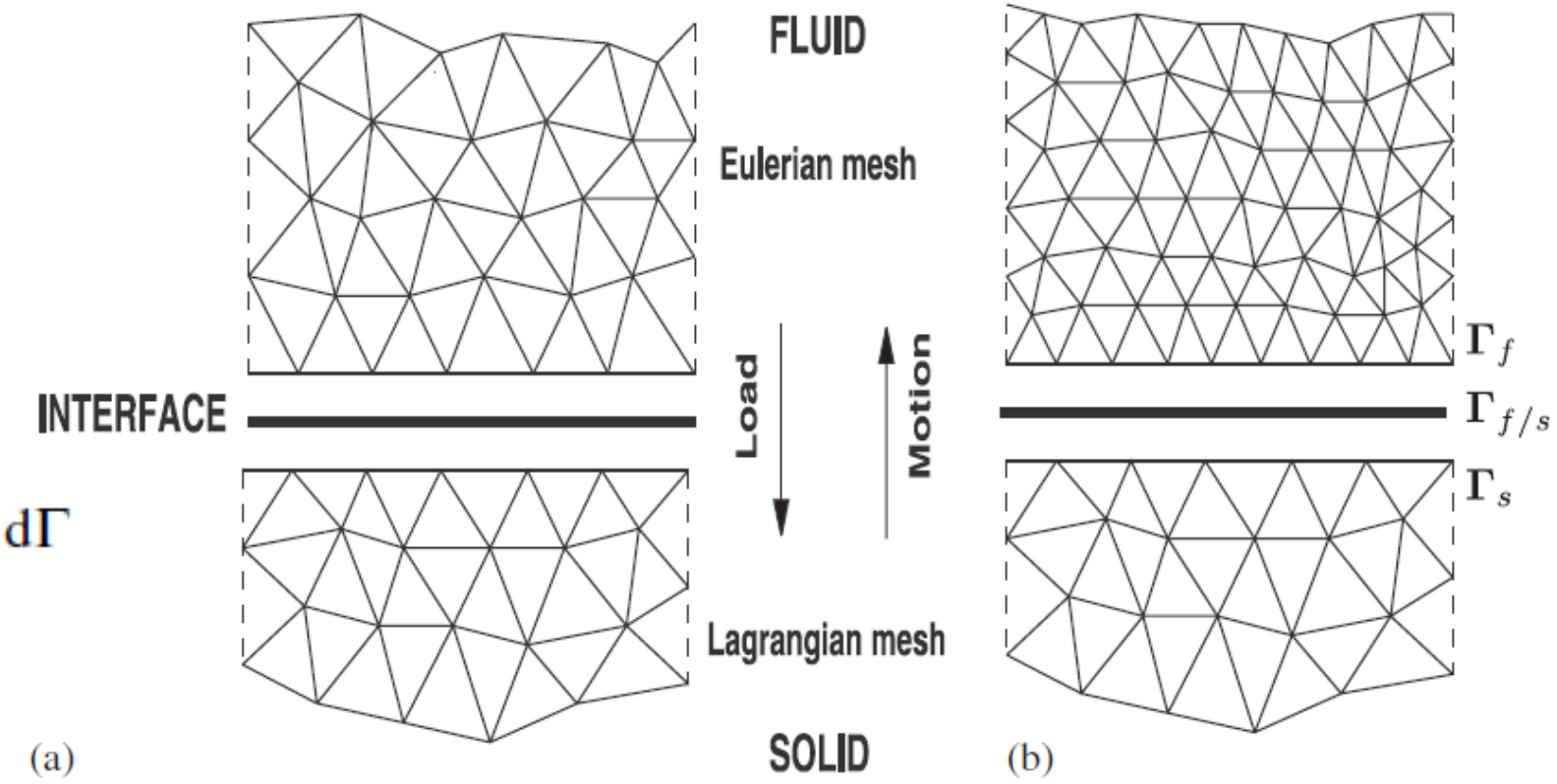
- Challenge:
  Conservation of …

1. Forces
   ➥Conservative

$$\mathscr{R}_s = \sum_{j=1}^{m_s} \mathscr{R}_s^j = \sum_{i=1}^{m_f} \mathscr{R}_f^i = \mathscr{R}_f$$

$$\mathscr{R}_s \equiv \int_\Gamma \mathbf{t}_s \, d\Gamma \qquad \mathscr{R}_f \equiv \int_\Gamma \mathbf{t}_f \, d\Gamma$$

2. Energy
   ➥Consistent

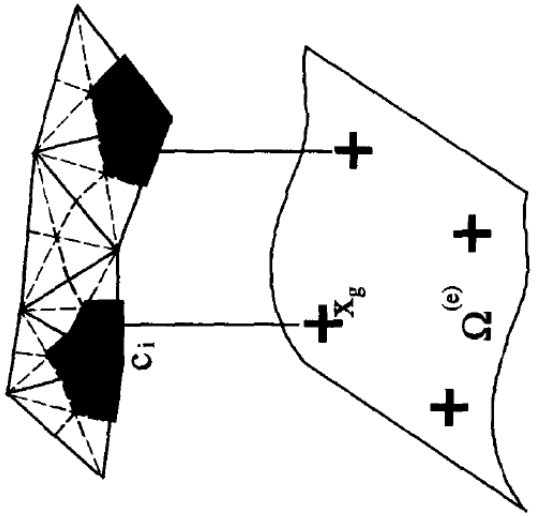


Jaiman et al (2005)

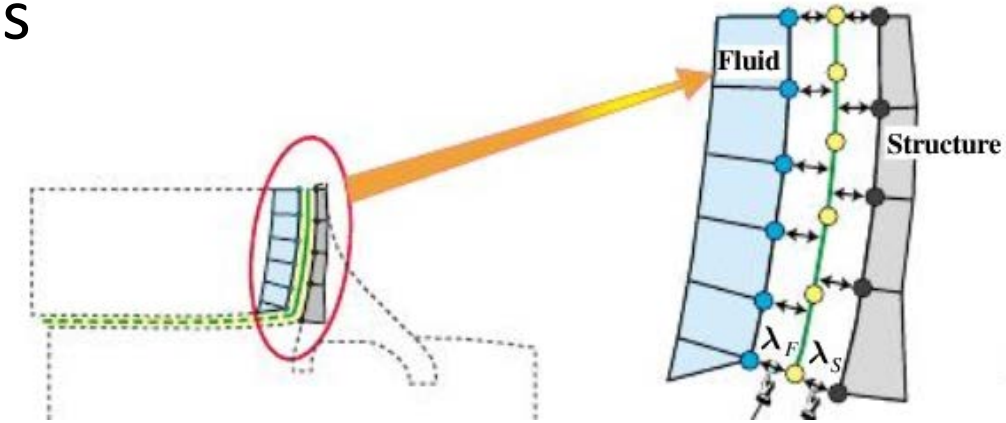
Interface data transfer

- Two families of procedures:

    1. Point match methods



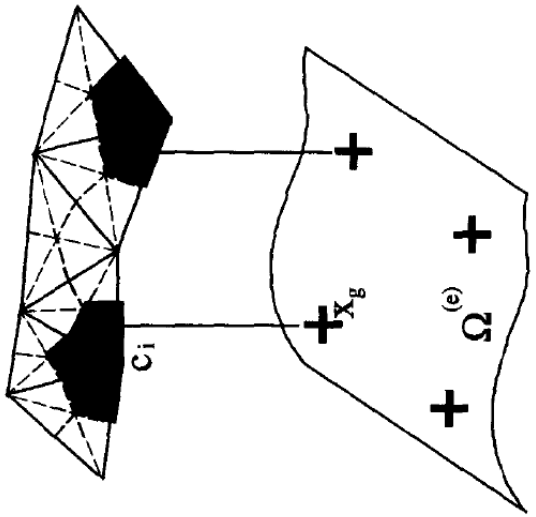
    2. Artificial interface methods



Interface data transfer

- Two families of procedures:

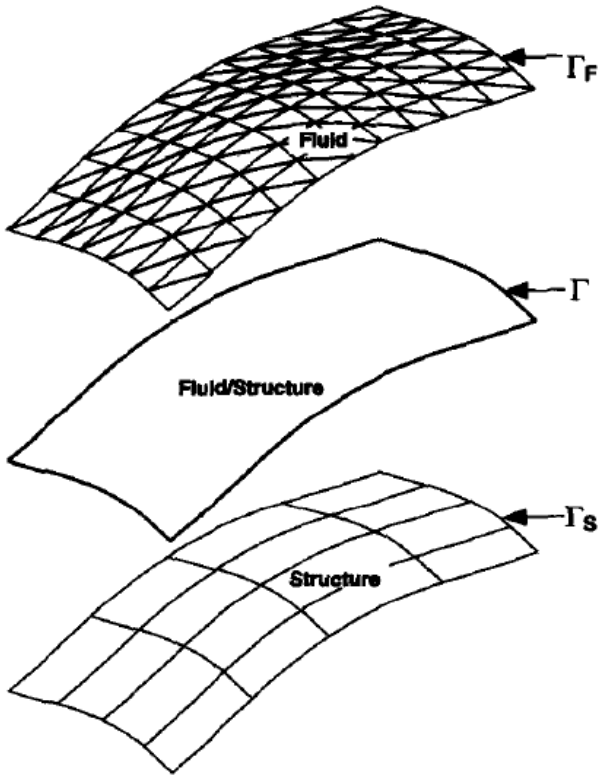
    1. Point match methods



    2. Artificial interface methods

Point match methods

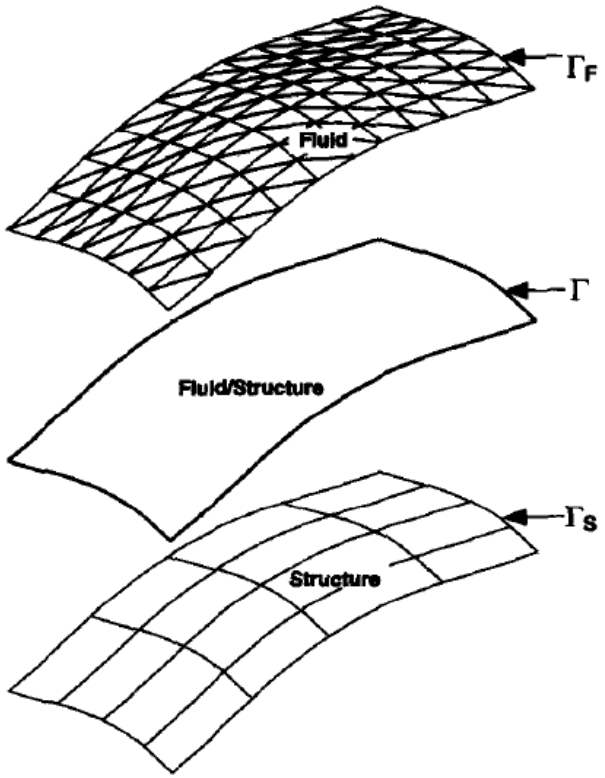
- Identify and match F/ and S/ points on the F/S itf
  - *Points …* Vertex, Gauss point, center of elements …
  - *Connection …* Shortest distance, normal projection …



Fahrat et al (1998)

Point match methods

- Identify and match F/ and S/ points on the F/S itf
  - *Points …* Vertex, Gauss point, center of elements …
  - *Connection …* Shortest distance, normal projection …
- Displacement: S→F
  - $\partial S_i \rightarrow \partial F_I$: rigid connectors
  - $\partial F_i \rightarrow \partial F_{J \neq I}$: local/global interpolation



Fahrat et al (1998)

Point match methods

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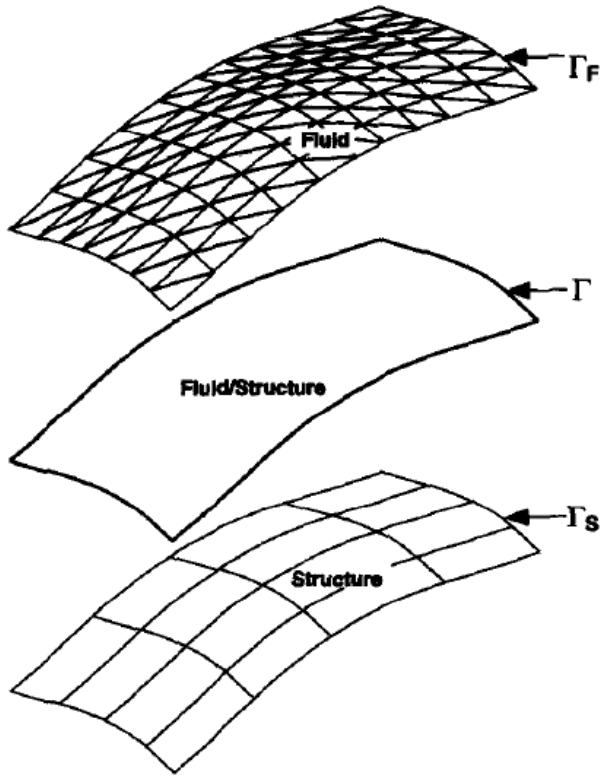
- Displacement: S→F
  - $\partial S_i \rightarrow \partial F_I$: rigid connectors
  - $\partial F_i \rightarrow \partial F_{J \neq I}$: local/global interpolation
- Pressure: F→S
  - Load is conserved (conservative)
  - Work is conserved (consistent)

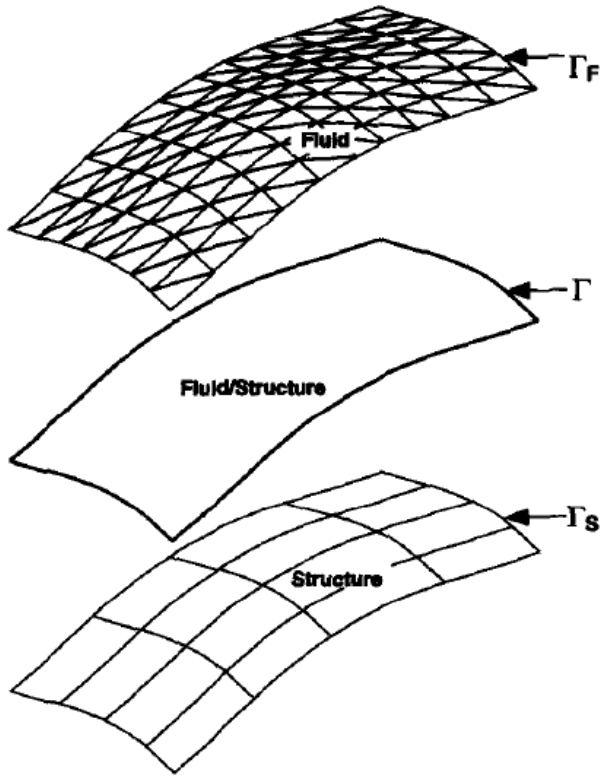


Fahrat et al (1998)

Point match methods

- Identify and match F/ and S/ points on the F/S itf
  - *Points …* Vertex, Gauss point, center of elements …
  - *Connection …* Shortest distance, normal projection …

1. "Exact" approach
2. "Approximate" approach
3. Non-conservative methods
4. Conservative methods

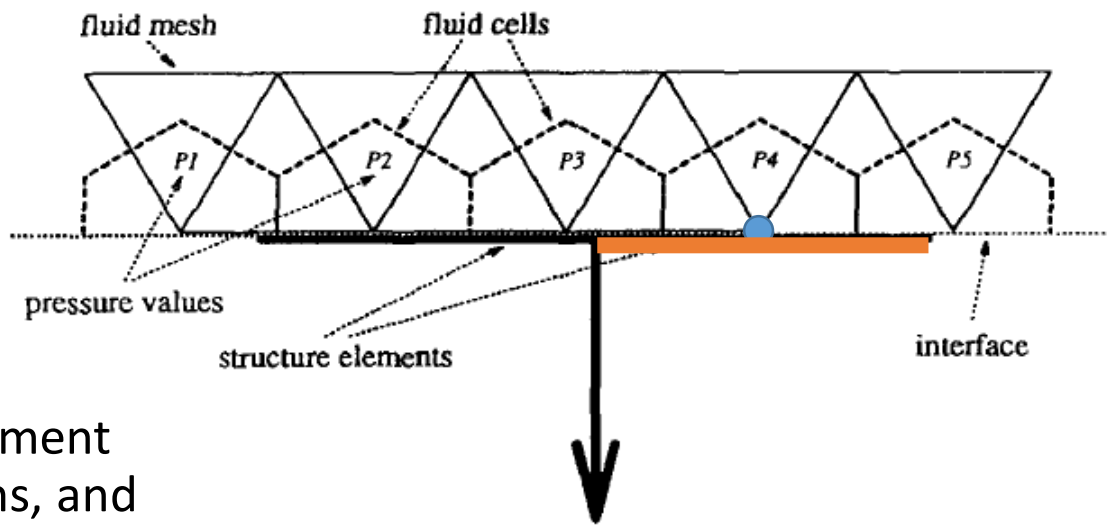


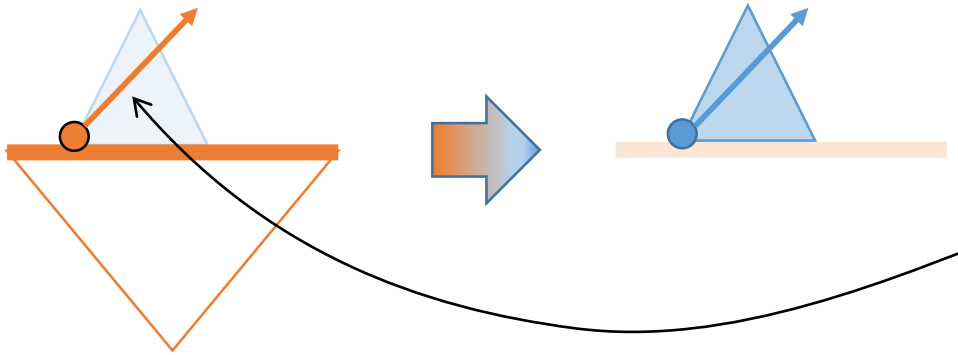
Fahrat et al (1998)

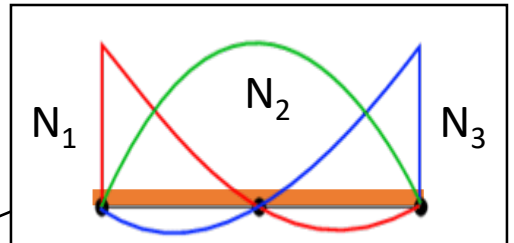
Point match methods

1/ “Exact” approach

- Displacements (S→F)
  - For each F/ point on the (F/) interface,
  - How to do it? ⇦ Find the associate S/ element,
  - Compute the displacement of the S/ element at that point through the shape functions, and
  - Assign this value to the F/ point







Structure element and shape functions

Maman & Fahrat (1995)

Point match methods

1/ “Exact” approach

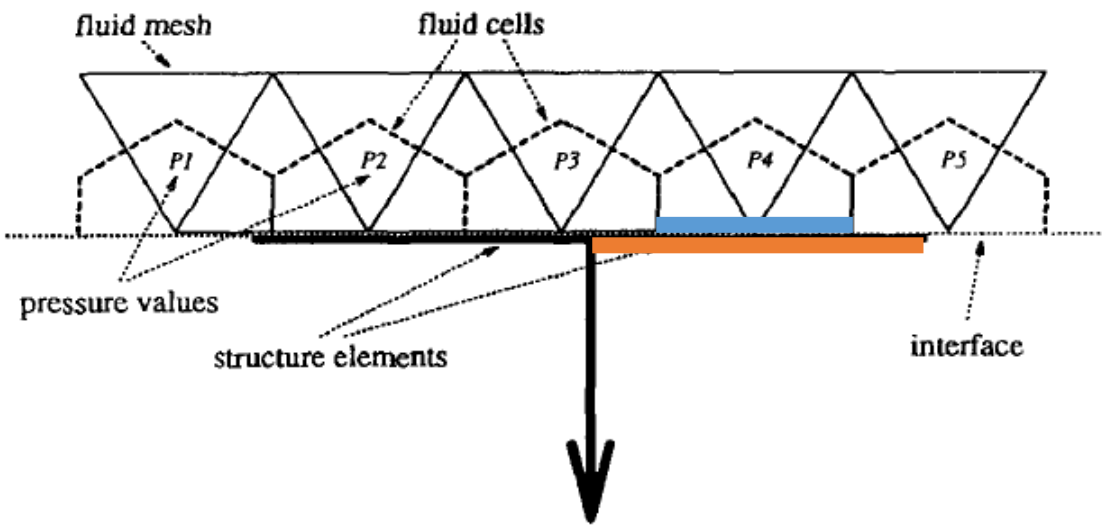
- Pressure loads (F→S)
  - For each F/ cell on the (F/) interface, get the boundary pressure load:

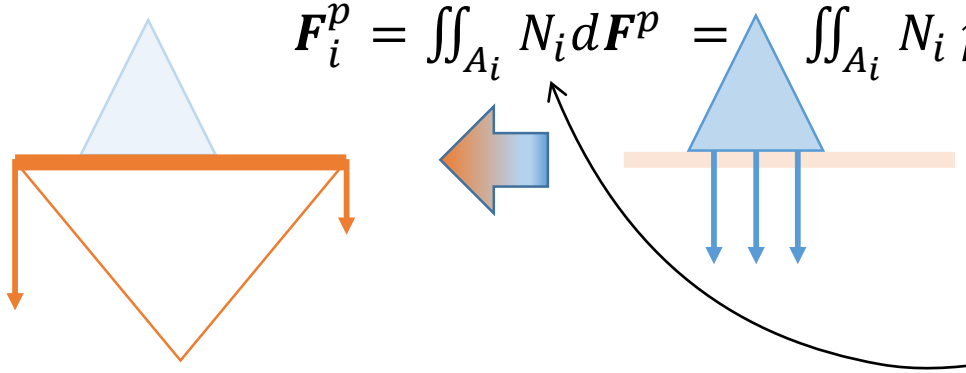
$$d\boldsymbol{F}^p = p\,\boldsymbol{n}\,dA$$

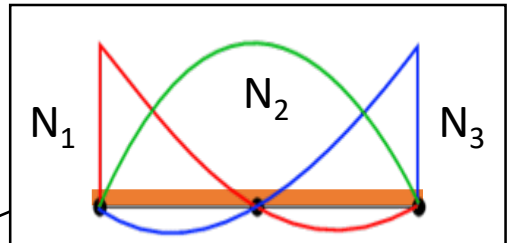
How to do it? ⇦

  - Find the associate S/ element(s), and
  - Compute the nodal loads through the S/ shape functions:

$$\boldsymbol{F}_i^p = \iint_{A_i} N_i d\boldsymbol{F}^p = \iint_{A_i} N_i\, p\,\boldsymbol{n}\,dA$$







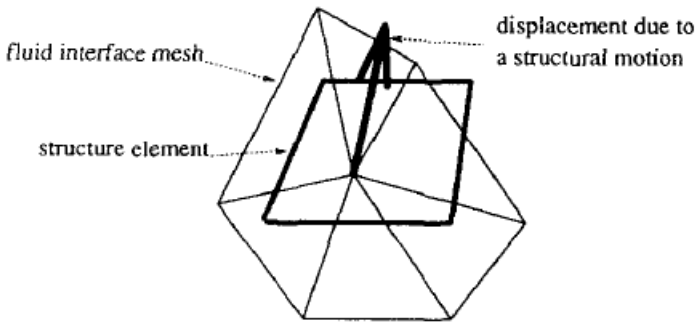
Structure element and shape functions

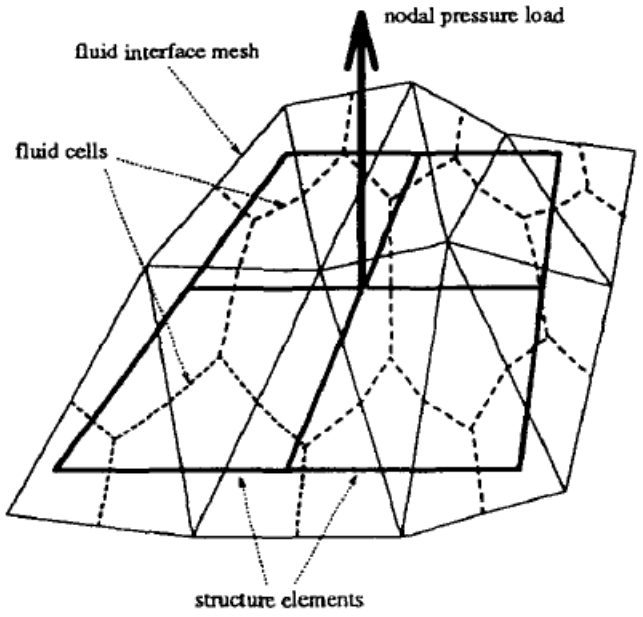
Maman & Fahrat (1995)

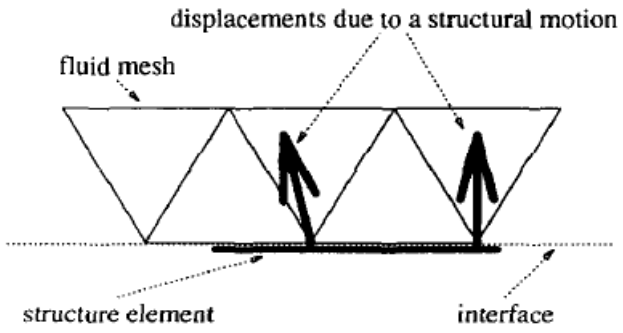
Point match methods

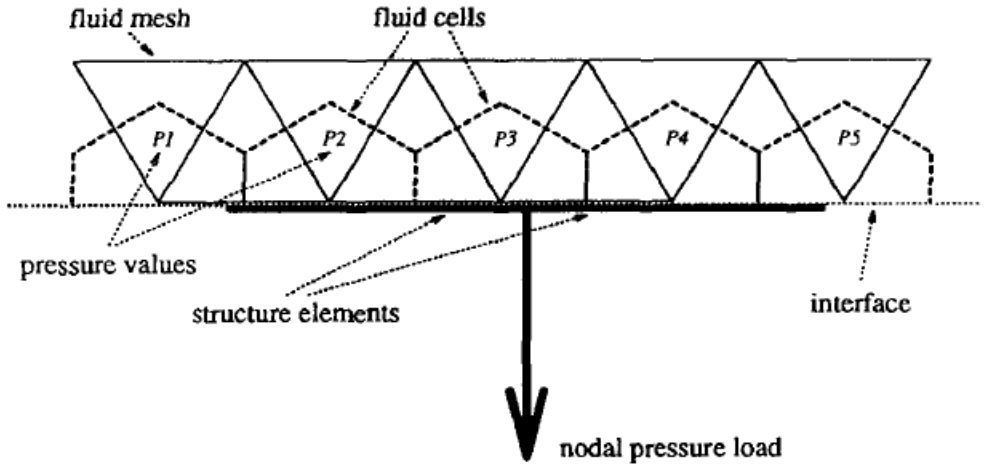
1/ “Exact” approach

- Benefits
  - Fast & accurate
  - 2D & 3D







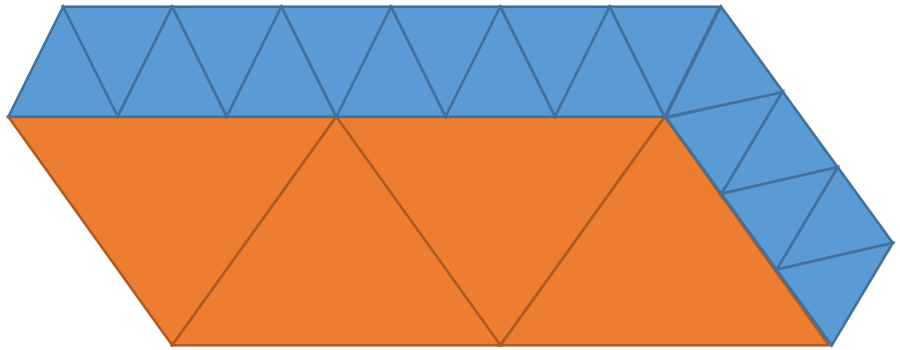


Maman & Fahrat (1995)

Point match methods

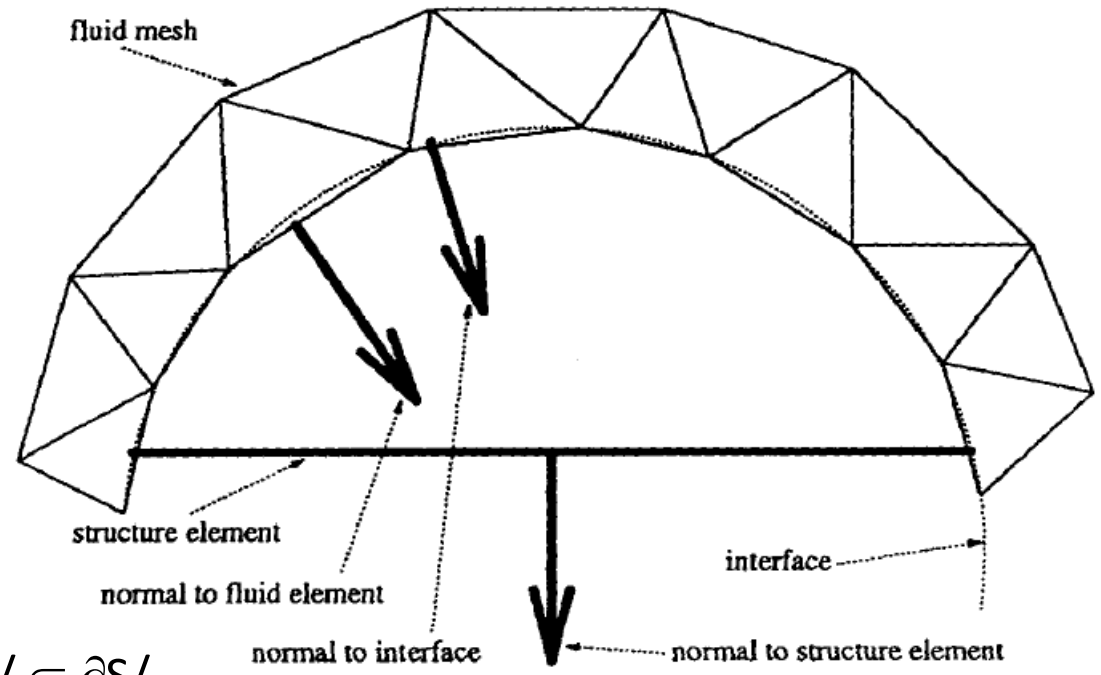
1/ “Exact” approach

- Benefits
  - Fast & accurate
  - 2D & 3D
- Drawbacks
  - Only for piecewise flat interfaces with $\partial F/ \subseteq \partial S/$



Maman & Fahrat (1995)

Point match methods



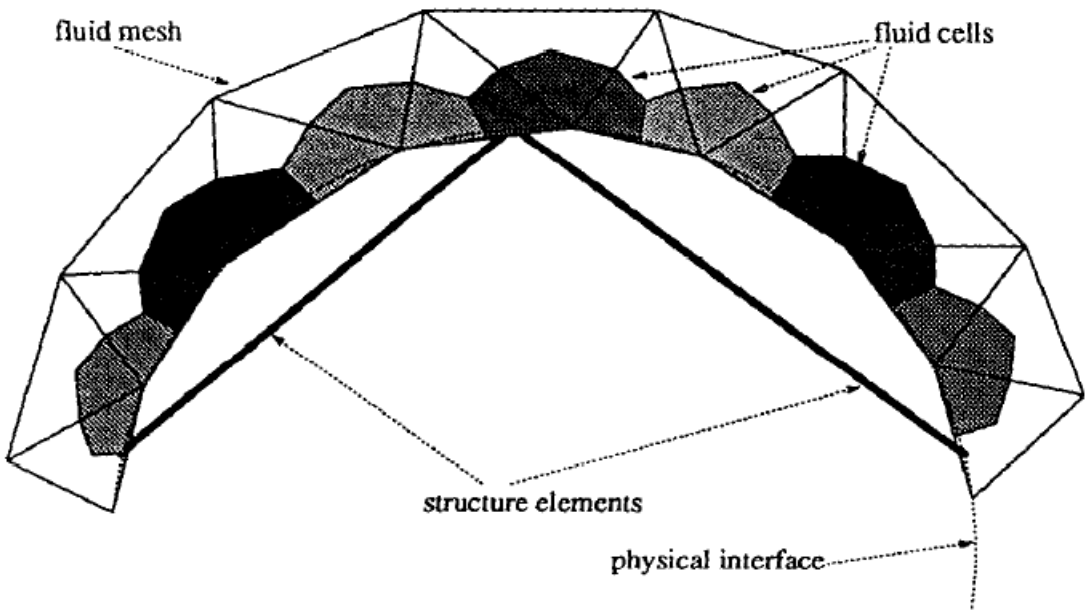
1/ “Exact” approach

- Benefits
  - Fast & accurate
  - 2D & 3D
- Drawbacks
  - Only for piecewise flat interfaces with $\partial F/ \subseteq \partial S/$
  - Which normal to compute the nodal pressure loads?

$$\boldsymbol{F}_i^p = \iint_{A_i} N_i \, p \, \boldsymbol{n} \, dA$$

Maman & Fahrat (1995)

Point match methods

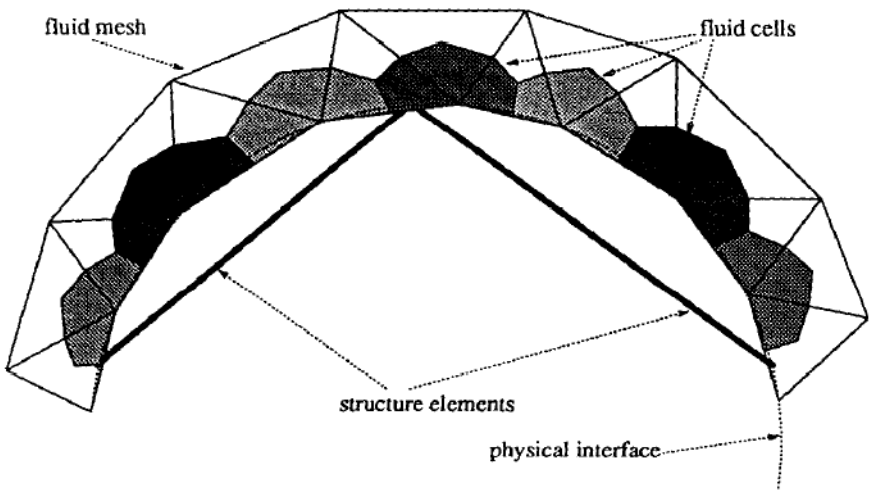


1/ “Exact” approach

- Benefits
  - Fast & accurate
  - 2D & 3D
- Drawbacks
  - Only for piecewise flat interfaces with $\partial F/ \subseteq \partial S/$
  - Which normal to compute the nodal pressure loads?
  - Intersection between F/ cells and S/ element might be $\varnothing$

Maman & Fahrat (1995)

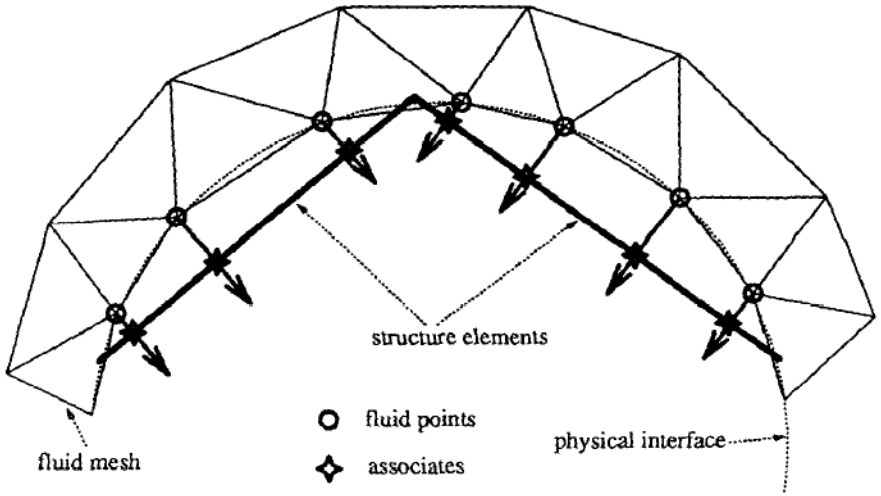
Point match methods



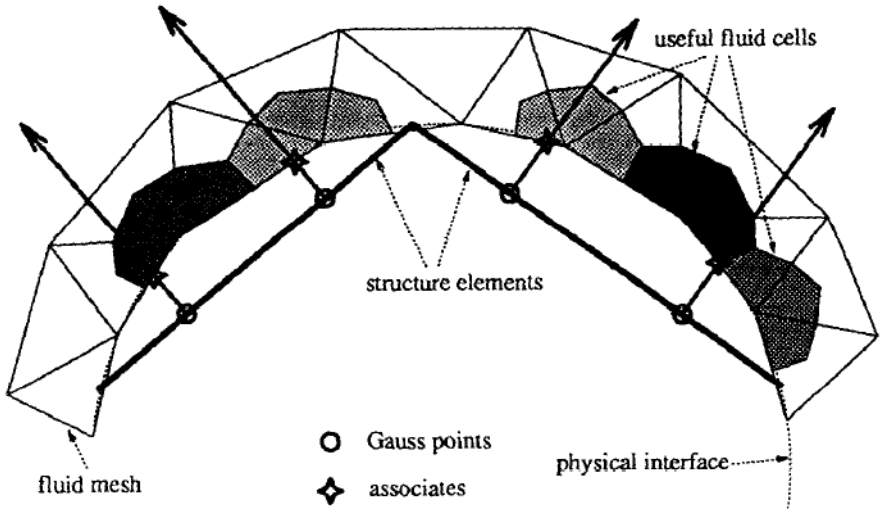
2/ “Approximate” approach

- Physical itf ≠ F/itf ≠ S/itf
- Choose a reference itf frame, e.g. S/itf
  - ➥ Project the F/itf onto the S/itf → *Associate points*

Displacements of F/ nodes



Pressure loads in S/ Gauss points



Maman & Fahrat (1995)

Point match methods

2/ “Approximate” approach

- Physical itf ≠ F/itf ≠ S/itf
- Choose a reference itf frame, e.g. S/itf
  - ➥ Project the F/itf onto the S/itf → *Associate* points
- Displacements: interpolation

  For each $y_j \in \partial F_m$ …

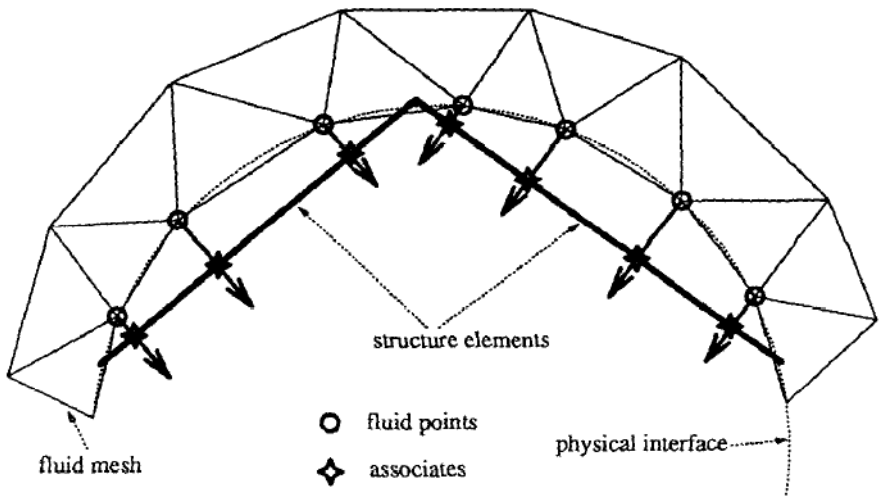
  → find associate S/element ∂S_n

  → find associate S/point $\bar{x}_j \in \partial S_n$

  → interpolate

  $$\boldsymbol{u}_F(y_j) = \boldsymbol{u}_S(\bar{x}_j) = \sum_i N_i(\bar{x}_j)\,\bar{\boldsymbol{u}}_i$$

Displacements of F/ nodes



Maman & Fahrat (1995)

Point match methods

2/ "Approximate" approach

- Physical itf ≠ F/itf ≠ S/itf
- Choose a reference itf frame, e.g. S/itf
  - ➥ Project the F/itf onto the S/itf → *Associate* points
- Pressure loads:

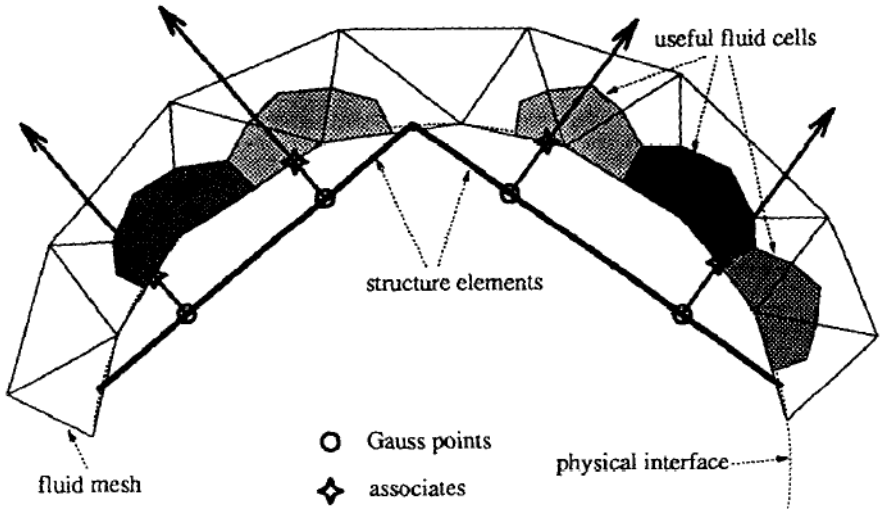
$$\boldsymbol{F}_i^p = \iint_{A_i} N_i \, p \, \boldsymbol{n} \, dA \approx \sum_{x_g} W_g \, (N_i \, p \, \boldsymbol{n})|_{x_g}$$

$N_i(x_g)$ … straightforward

$p(x_g)$ ← pressure in the F/point associate to x_g

$\boldsymbol{n} = \boldsymbol{n}(x_g)$ if curved S/ bnd.

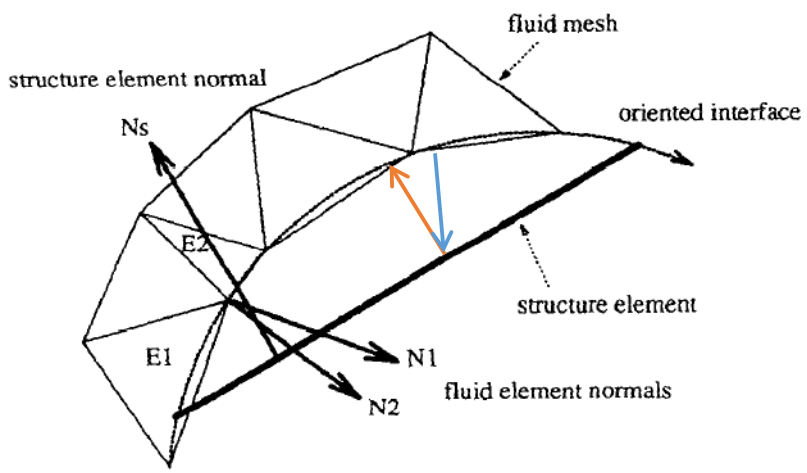
Pressure loads in S/ Gauss points



Maman & Fahrat (1995)

Point match methods

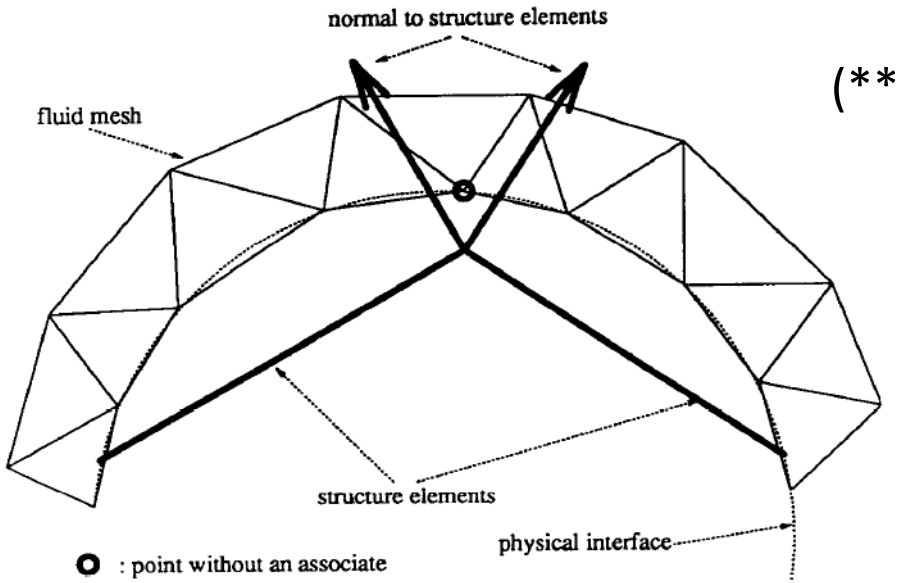
(*)



2/ "Approximate" approach

- Physical itf. ≠ F/bnd. ≠ S/bnd.
- Choose a reference itf frame, e.g. S/bnd
  - ➥ Project the F/itf. onto the S/itf. → Associate points
- Pressure loads: $\boldsymbol{F}_i^p = \sum_{x_g} W_g \left(N^i \, p \, \boldsymbol{n}\right)|_{x_g}$
- Normals…
  - Ambiguity (*)
    ➥ $\boldsymbol{n} \perp$ S/bnd
  - No associates (**)

(**)



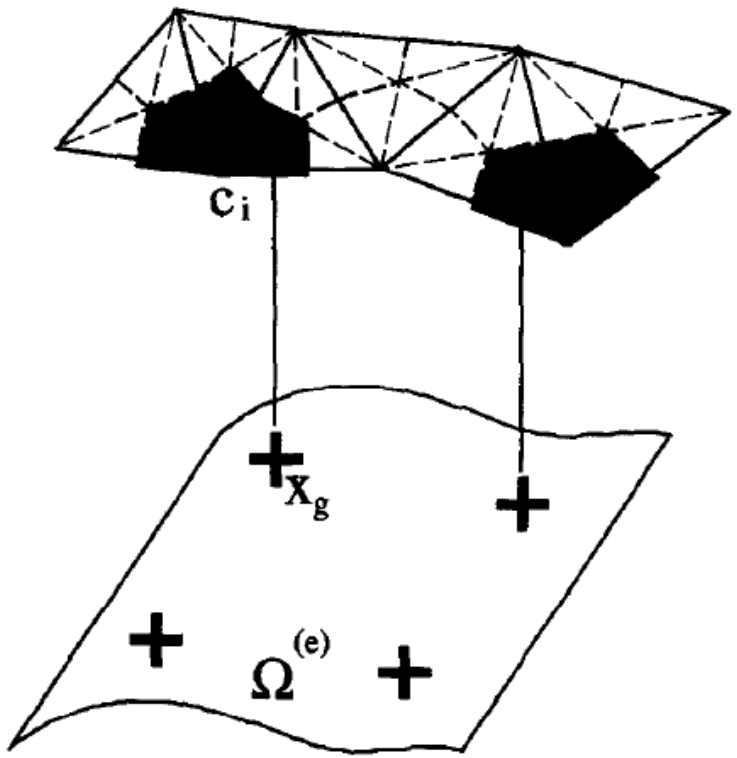
Maman & Fahrat (1995)

Point match methods

3/ Non-conservative method

$$f_i^{(e)} = \int_{\Omega_S^{(e)}} N_i(-pn + \sigma_F.n)\,\mathrm{d}s \quad = \sum_{g=1}^{g=n_g} w_g N_i(X_g)(-p(X_g)n + \sigma_F(X_g).n)$$

- Need $p(X_g)$ and $\sigma_F(X_g)$
- Match X_g with a F/ cell or element
- Use the F/ shape functions to interpolate p and σ_F in X_g
  (consistent interpolation method)



Fahrat et al (1998)

Point match methods

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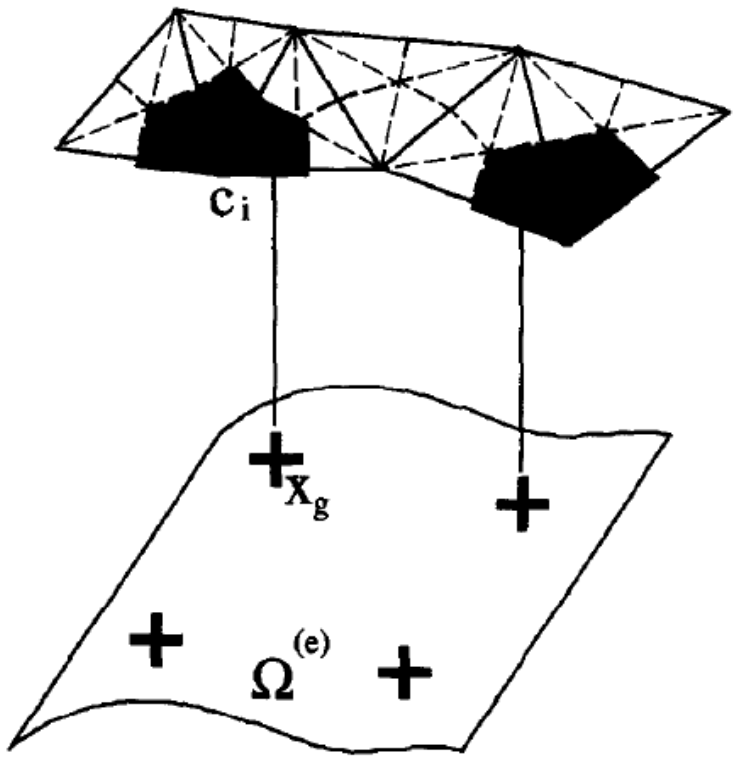


- Need $p(X_g)$ and $\sigma_F(X_g)$
- Match X_g with a F/ cell or element
- Use the F/ shape functions to interpolate p and σ_F in X_g

- Remarks
  - $f_i^{(e)}$ may include both forces and moments
  - Forces are *not* conserved through the F-S interface …
    - ➥ Not a big issue as far as $\Gamma_F \sim \Gamma_S$
    - ➥ Add Gauss points to improve conservation

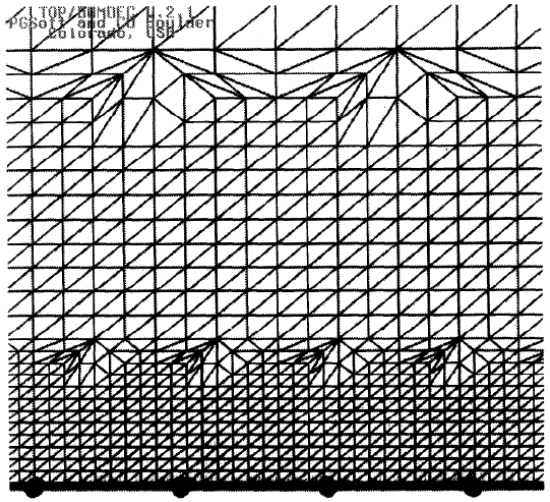
Fahrat et al (1998)

Point match methods

3/ Non-conservative method

- Example: Flat panel in a supersonic stream, transient response

F: inviscid fluid, 8k elems



S: Euler beam, 10 elems, 2 GP/elem

Lift

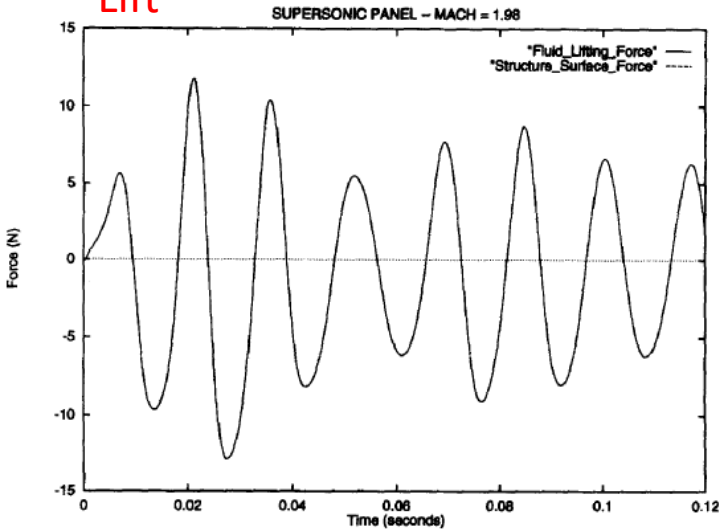


Fig. 5. Aerodynamic lifting force vs. structural surface force.

Energy

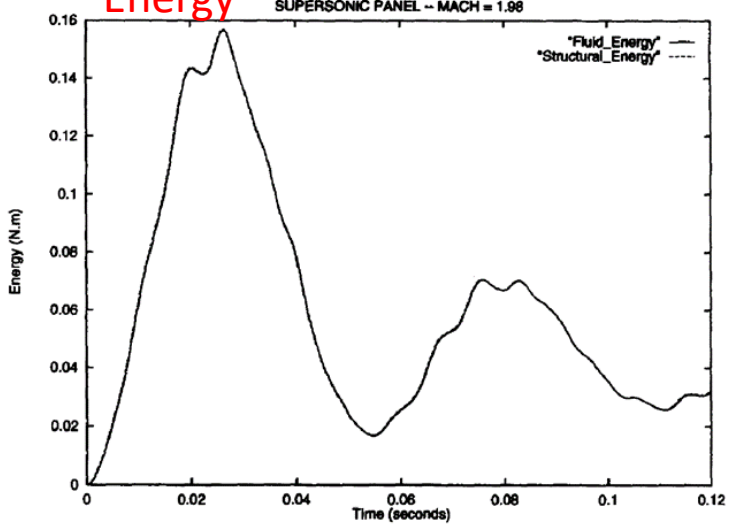


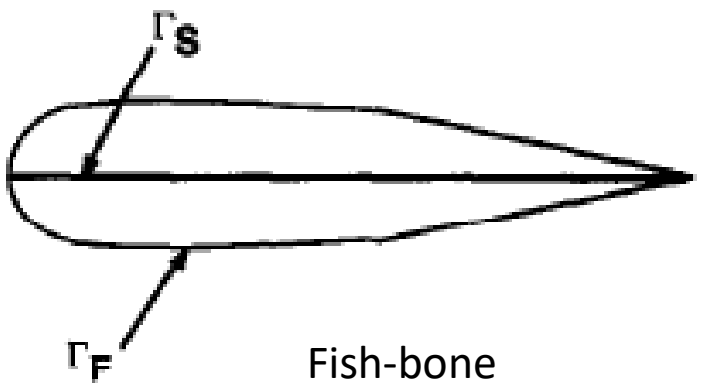
Fig. 6. Energy gained/released by the fluid vs. energy released/gained by the structure.

Fahrat et al (1998)

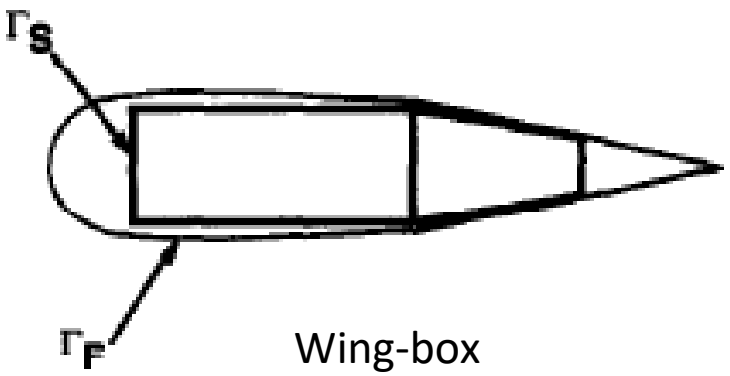
Point match methods

3/ Non-conservative method

- Drawback



Fish-bone



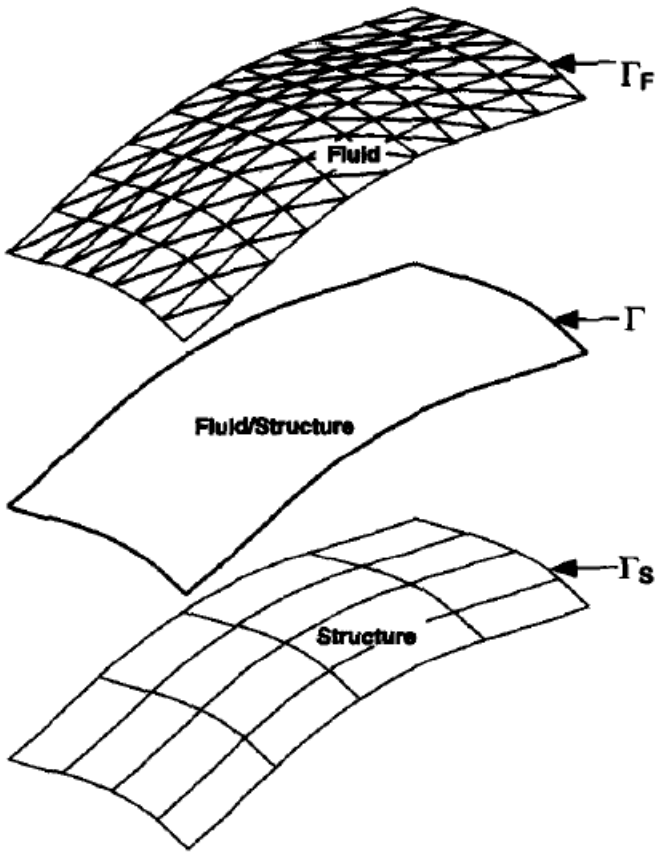
Wing-box

Fahrat et al (1998)

Point match methods

4/ Conservative method

- Admissible virtual displacements: $\hat{\boldsymbol{u}}_F$ and $\hat{\boldsymbol{u}}_S$
  - Continuum: $\hat{\boldsymbol{u}}_F = \hat{\boldsymbol{u}}_S$ on Γ
  - Discrete: $\hat{\boldsymbol{u}}_{Fj} = \sum_{i=1}^{i_S} c_{ji} \hat{\boldsymbol{u}}_{Si}$, $j \in \Gamma_F$, $i \in \Gamma_S$
- Interpolation of virtual fluid field displacement with support on Γ_F:
  - $\hat{\boldsymbol{u}}_F = \sum_{j=1}^{j_F} D_j \hat{\boldsymbol{u}}_{Fj}$, $j \in \Gamma_F$



Fahrat et al (1998)

Point match methods

4/ Conservative method

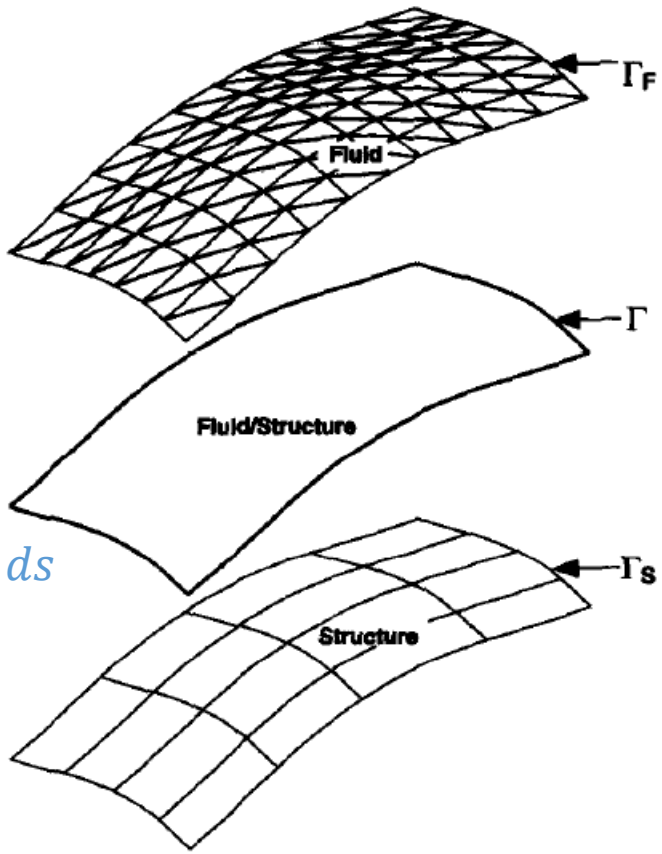
- Admissible stress tensors: σ_F and σ_S
  - Continuum: $\sigma_S \cdot \boldsymbol{n} = -p\,\boldsymbol{n} + \tau \cdot \boldsymbol{n}$ on Γ
- Virtual work on Γ

$$\Phi_j = \int_{\Gamma_F} (-p\,\boldsymbol{n} + \tau \cdot \boldsymbol{n})\, D_j\, ds$$

  - $\delta W^F = \int_{\Gamma_F} (-p\,\boldsymbol{n} + \tau \cdot \boldsymbol{n}) \cdot \hat{\boldsymbol{u}}_F\, ds = \cdots = \sum_{j=1}^{j_F} \Phi_j \hat{\boldsymbol{u}}_{Fj}$

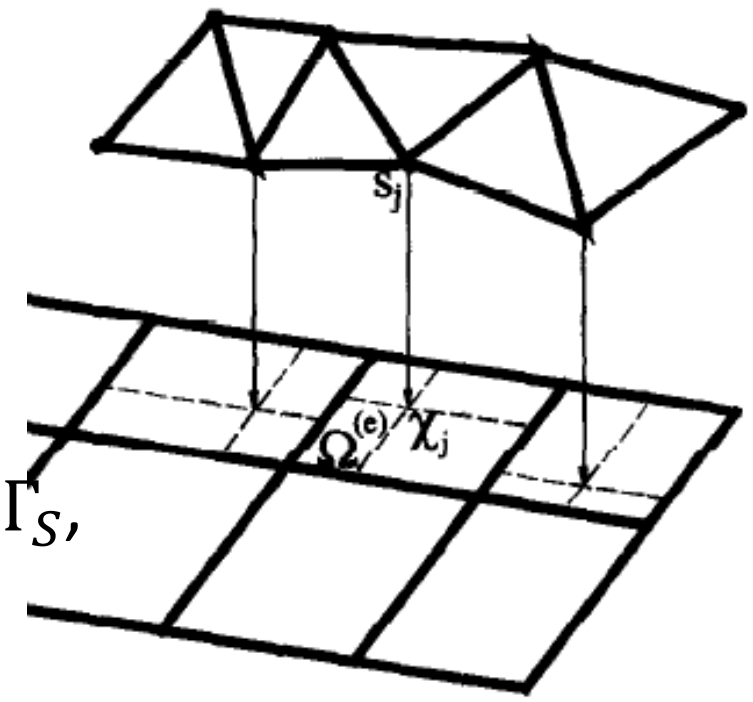
    $= \sum_{i=1}^{i_S} \left(\sum_{j=1}^{j_F} \Phi_j c_{ji} \right) \hat{\boldsymbol{u}}_{Si} = \sum_{i=1}^{i_S} f_i\, \hat{\boldsymbol{u}}_{Si} = \delta W^S$

    → $f_i = \sum_{j=1}^{j_F} \Phi_j c_{ji}$ … nodal forces on the S/ mesh
- Remark: f_i does not depend on the S/ shape functions



Fahrat et al (1998)

Point match methods



4*/ Conservative method & Consistent interpolation

- S/ displacement field: $\boldsymbol{u}_S^{(e)} = \sum_{i=1}^{i_e} N_i \, \boldsymbol{u}_{Si}$
- (i) pairing each fluid grid point $S_j \in \Gamma_F$ with $\Omega_S^{(e)} \in \Gamma_S$,
  (ii) determining the natural coordinates χ_j in $\Omega_S^{(e)}$
  of the fluid point S_j (or its projection onto $\Omega_S^{(e)}$)
  (iii) interpolating $\boldsymbol{u}_F$ inside $\Omega_S^{(e)}$ using the S/ shape functions:

$$\boldsymbol{u}_{Fj} = \boldsymbol{u}_F(S_j) = \boldsymbol{u}_S(\chi_j) = \sum_{i=1}^{i_e} N_i(\chi_j)\boldsymbol{u}_{Si}$$

  (iv) recall: $\widehat{\boldsymbol{u}}_{Fj} = \sum_{i=1}^{i_S} c_{ji}\widehat{\boldsymbol{u}}_{Si}$, $f_i = \sum_{j=1}^{j_F} \Phi_j c_{ji}$

➥ $f_i = \sum_{j=1}^{j_F} \Phi_j N_i(\chi_j)$ with $\sum_{i=1}^{i_S} f_i = \sum_{j=1}^{j_F} \Phi_j$

Fahrat et al (1998)

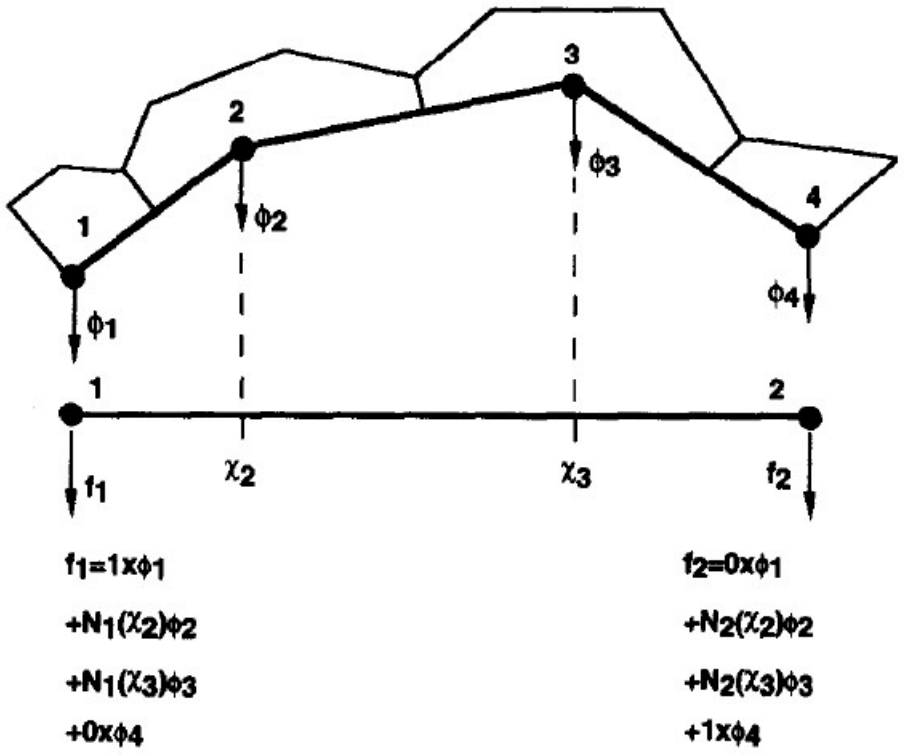
Point match methods

4*/ Conservative method & Consistent interpolation

- Check conservation of forces

$$
\begin{aligned}
f_1 &= N_1(\chi_1)\Phi_1 + N_1(\chi_2)\Phi_2 + N_1(\chi_3)\Phi_3 + N_1(\chi_4)\Phi_4 \\
&= \Phi_1 + N_1(\chi_2)\Phi_2 + N_1(\chi_3)\Phi_3 \\
f_2 &= N_2(\chi_1)\Phi_1 + N_2(\chi_2)\Phi_2 + N_2(\chi_3)\Phi_3 + N_2(\chi_4)\Phi_4 \\
&= N_2(\chi_2)\Phi_2 + N_2(\chi_3)\Phi_3 + \Phi_4
\end{aligned}
$$

$$
\begin{aligned}
f_1 + f_2 &= \Phi_1 + (N_1(\chi_2) + N_2(\chi_2))\Phi_2 + (N_1(\chi_3) + N_2(\chi_3))\Phi_3 + \Phi_4 \\
&= \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4
\end{aligned}
$$

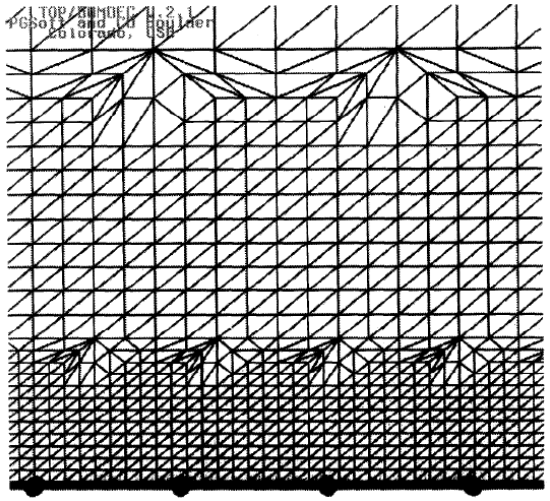


Fahrat et al (1998)

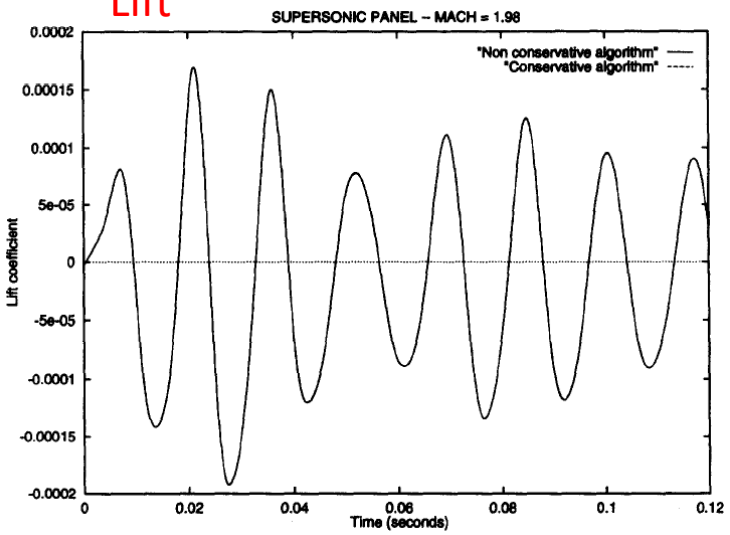
Point match methods

4*/ Conservative method & Consistent interpolation

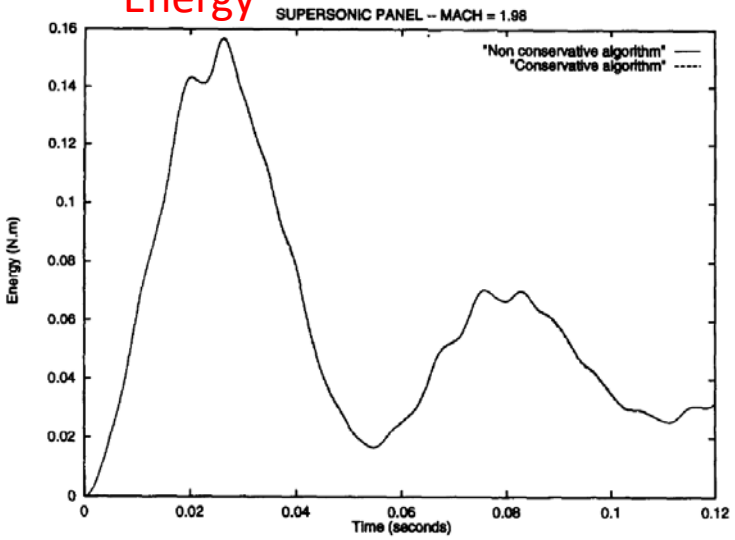
- Example:
  Flat panel in a supersonic stream



Lift



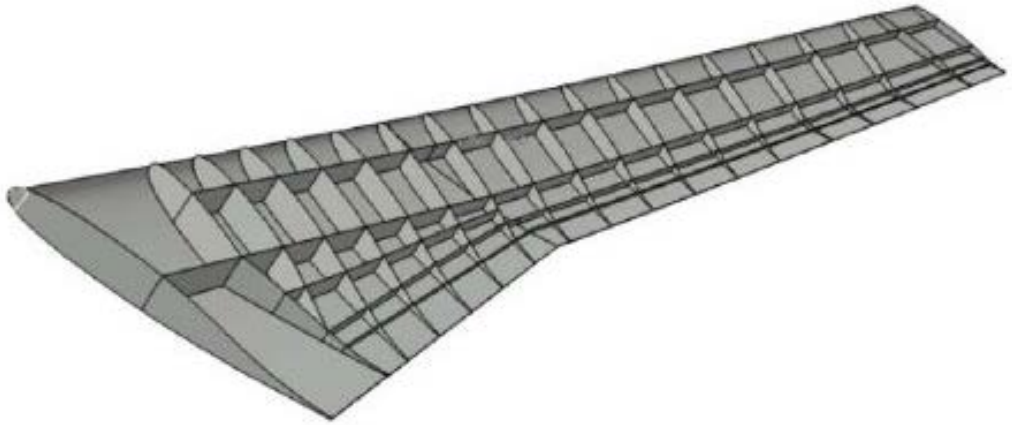
Energy



Fahrat et al (1998)

Point match methods

- Another example: transient aeroelastic response of the ARW-2 wing in a transonic airstream

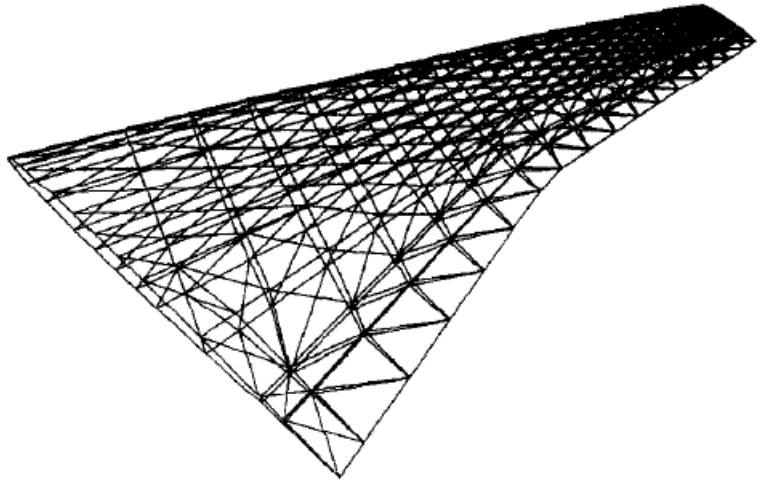


Fahrat et al (1998)

Point match methods

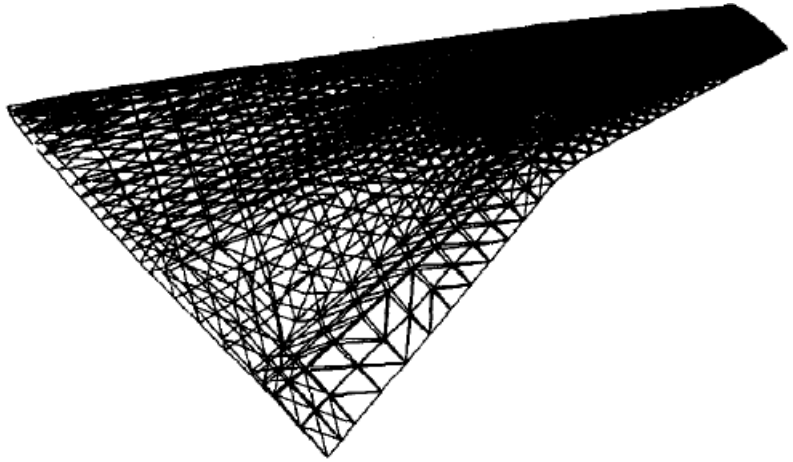
- Another example: transient aeroelastic response of the ARW-2 wing in a transonic airstream

S/ Surface mesh: >800 triangles



S/ Volume elements: ~300 bars, ~300 beams, ~800 membrane, >300 shell

F/ Surface mesh: <4000 triangles



F/ Volume mesh: ~160k tetrahedra

Fahrat et al (1998)

Point match methods

- Another example: transient aeroelastic response of the ARW-2 wing in a transonic airstream

Non-conservative method

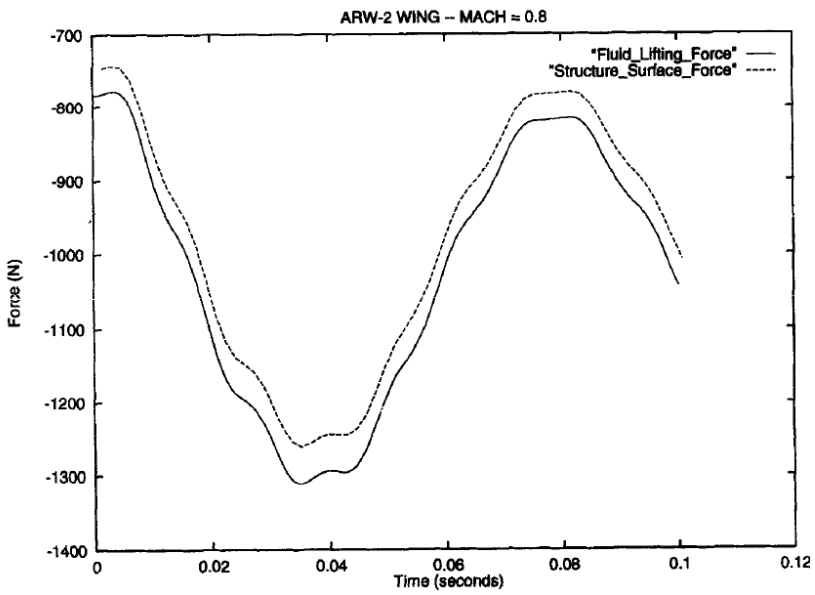


Fig. 18. Aerodynamic lifting force vs. structural surface force (consistent interpolation method).

Lift

Conservative method

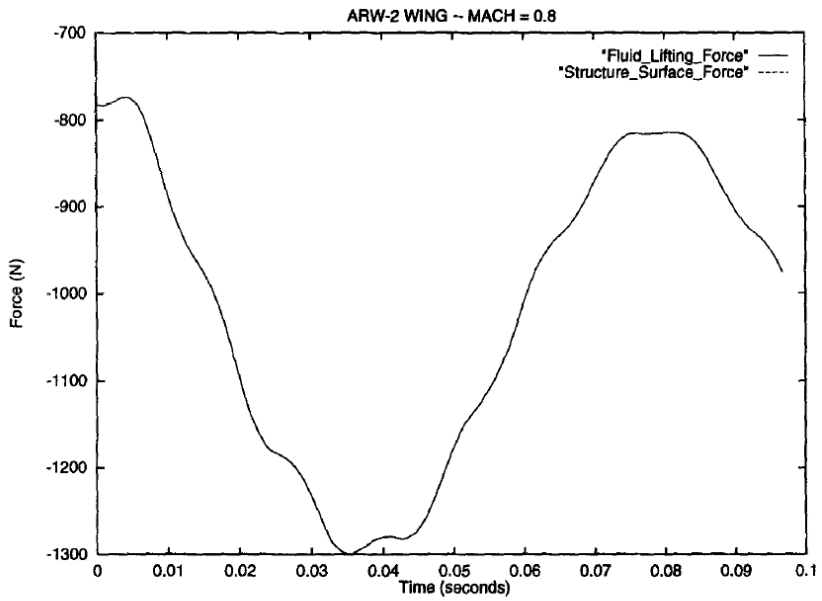


Fig. 20. Aerodynamic lifting force vs. structural surface force (new conservative method).

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Point match methods

- Another example: transient aeroelastic response of the ARW-2 wing in a transonic airstream

Non-conservative method

Energy

Conservative method

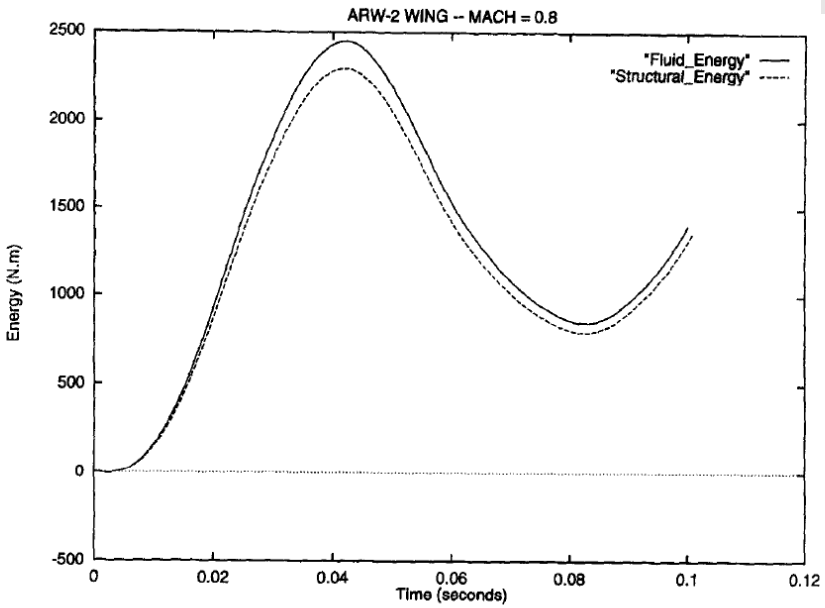

Fig. 19. Energy gained/released by the fluid vs. energy released/gained by the structure (consistent interpolation method).

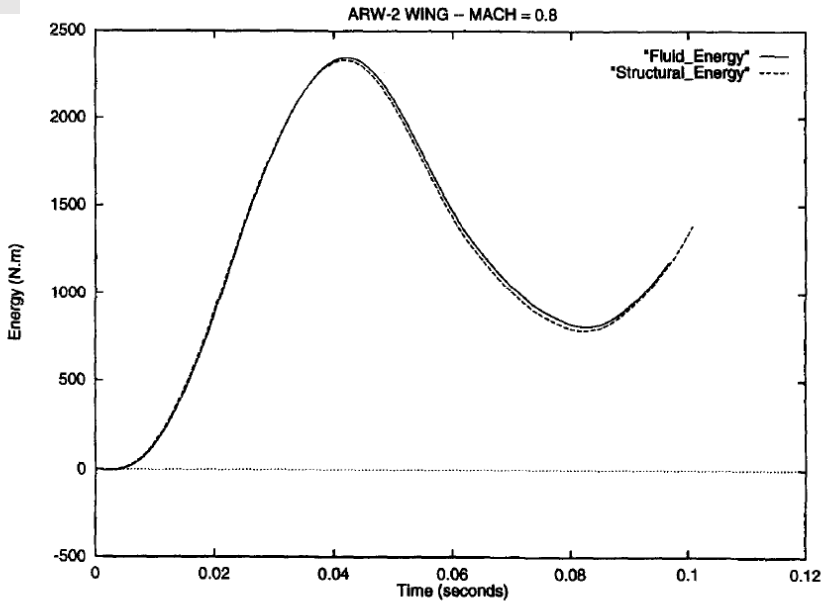

Fig. 21. Energy gained/released by the fluid vs. energy released/gained by the structure (new conservative method).

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